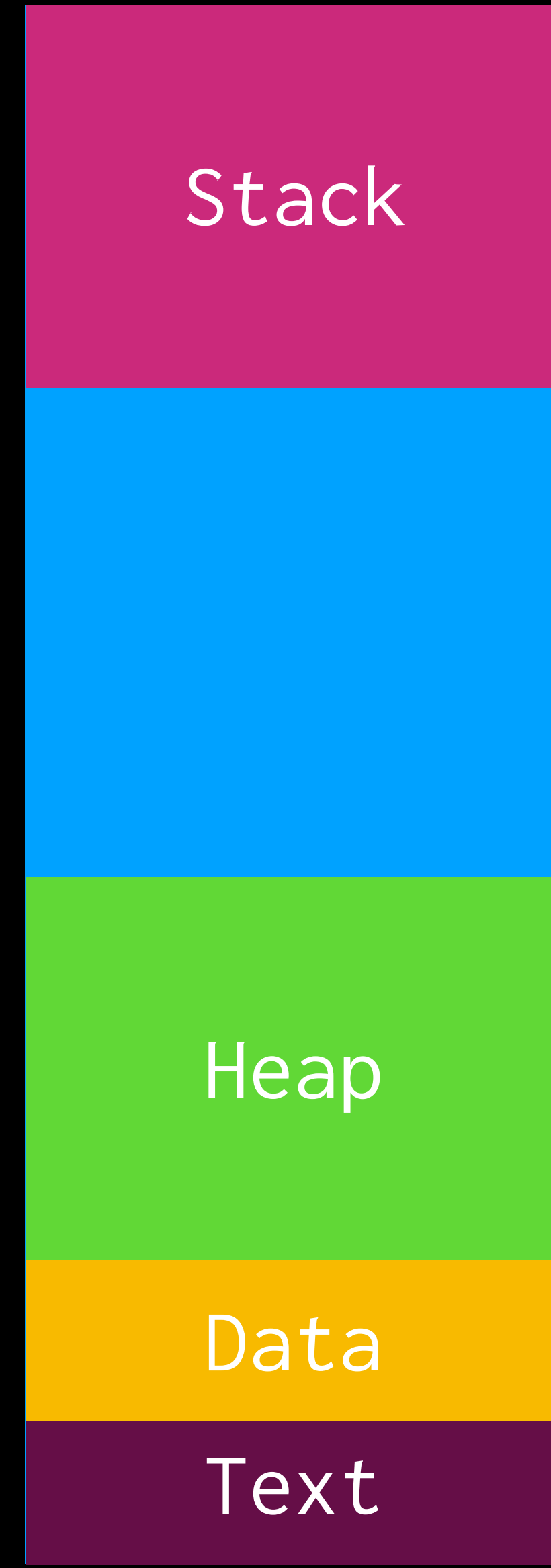
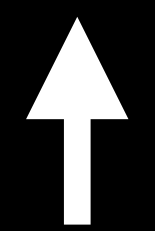
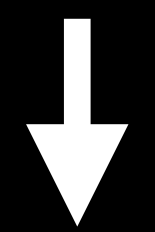


memory



Stack



Heap

Data

Text

Q: *how to manage memory?*

lexical scoping

```
struct foo {
    int x1;
    int x2;
};

void f(struct foo *sf);

int x = 1;
int z;
{
    struct foo sf;
    f(&sf);
    {
        int w = 1;
        sf.f2 = w;
        z += sf.f1;
    }
}
return z;
```

lexical scoping

```
struct foo {  
    int x1;  
    int x2;  
};  
  
void f(struct foo *sf);  
  
int x = 1;  
int z;  
{  
    struct foo sf;  
    f(&sf);  
    {  
        int w = 1;  
        sf.f2 = w;  
        z += sf.f1;  
    }  
}  
return z;
```

usage (B)

int x = 1;	+4
int z;	+4
{	
struct foo sf;	+8
f(&sf);	
{	
int w = 1;	+4
sf.f2 = w;	
z += sf.f1;	
}	-4
}	-8
return z;	-8

Stack



lexical scoping

```
struct foo {
    int x1;
    int x2;
};

void f(struct foo *sf);

int x = 1;
int z;
{
    struct foo sf;
    f(&sf);
    {
        int w = 1;
        sf.f2 = w;
        z += sf.f1;
    }
}
return z;
```

“stack discipline”

allocation is as easy as
decrementing stack pointer!

manual

```
struct foo {  
    int x1;  
    int x2;  
};
```

```
struct foo *f() {  
    return alloca(sizeof(struct foo));  
}
```

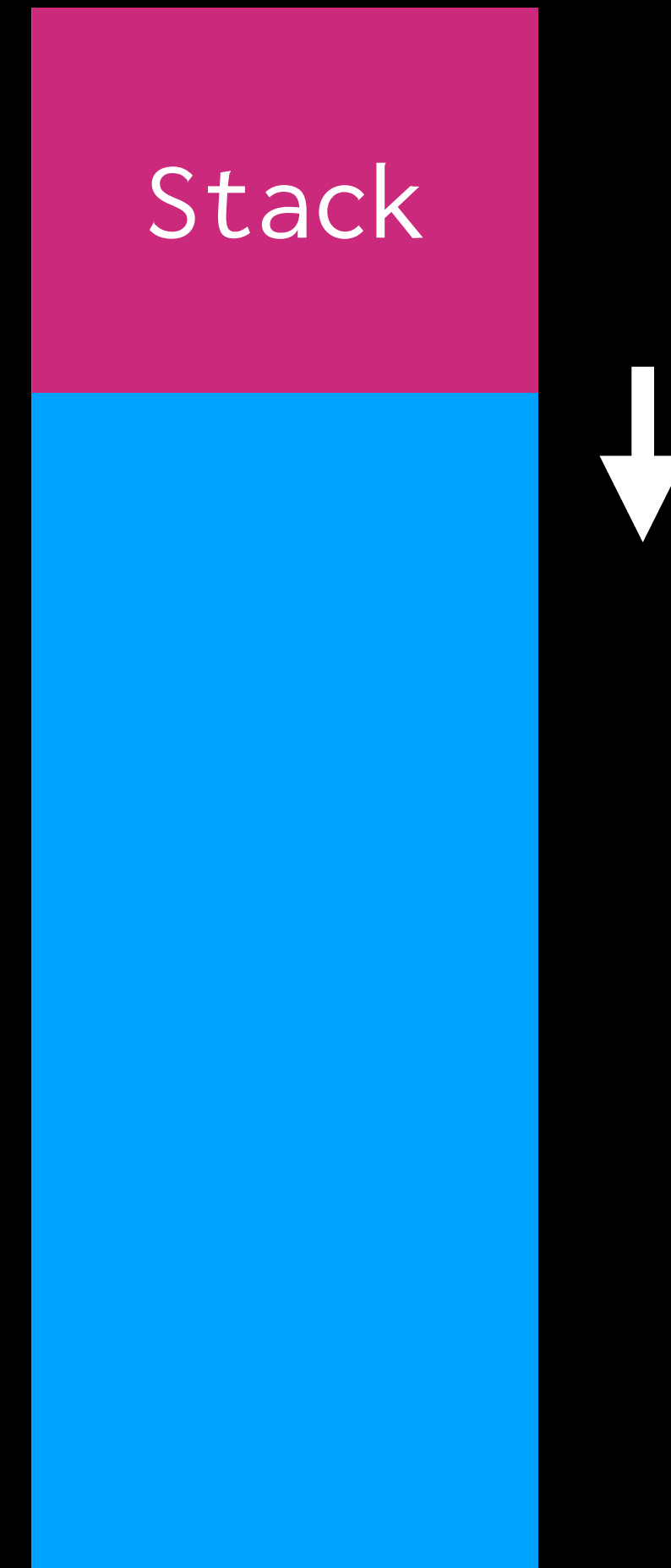
```
struct foo *sf = f();  
return sf->x1;
```

manual

```
struct foo {  
    int x1;  
    int x2;  
};
```

```
struct foo *f() {  
    return alloca(sizeof(struct foo));  
}
```

```
struct foo *sf = f();  
return sf->x1;
```



manual

```
struct foo {  
    int x1;  
    int x2;  
};
```

clobbered!

```
struct foo *f() {  
    return alloca(sizeof(struct foo));  
}
```

```
struct foo *sf = f();  
return sf->x1;
```

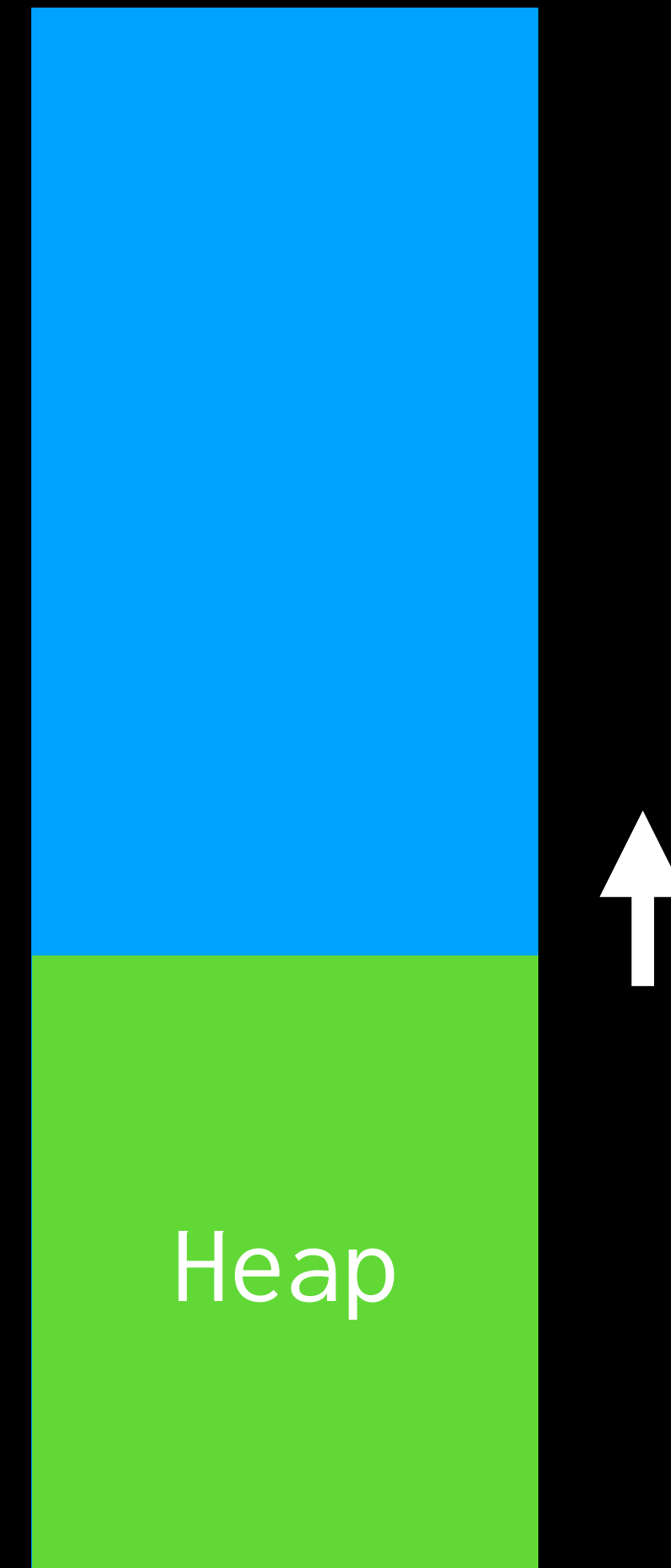


manual

```
struct foo {  
    int x1;  
    int x2;  
};
```

```
struct foo *f() {  
    return malloc(sizeof(struct foo));  
}
```

```
struct foo *sf = f();  
return sf->x1;
```



manual

```
struct foo {  
    int x1;  
    int x2;  
};
```

how is this implemented?

```
struct foo *f() {  
    return malloc(sizeof(struct foo));  
}
```

```
struct foo *sf = f();  
return sf->x1;
```

manual

```
struct foo {  
    int x1;  
    int x2;  
};
```

```
struct foo *f() {  
    return malloc(sizeof(struct foo));  
}
```

```
struct foo *sf = f();  
return sf->x1;
```

how much heap are we still using?

manual

```
struct foo {  
    int x1;  
    int x2;  
};
```

```
struct foo *f() {  
    return malloc(sizeof(struct foo));  
}
```

```
struct foo *sf = f();  
int retval = sf->x1;  
free(sf);  
return retval;
```

manual

```
struct foo {  
    int x1;  
    int x2;  
};
```

```
struct foo *f() {  
    return malloc(sizeof(struct foo));  
}
```

```
struct foo *sf = f();  
int retval = sf->x1;  
free(sf);  
return retval;
```

what a hassle!

reference counting

```
struct foo {
    int x1;
    int x2;
};

void f(struct foo *sf);

int x = 1;
int z;
{
    struct foo *sf = rc_alloc(sizeof(struct foo));
    f(sf);
    {
        int w = 1;
        sf.f2 = w;
        z += sf.f1;
    }
}
return z;
```

reference counting

```
struct foo {
    int x1;
    int x2;
};

void f(struct foo *sf);

int x = 1;
int z;
{
    struct foo *sf = rc_alloc(sizeof(struct foo));
    f(sf);
    {
        int w = 1;
        sf.f2 = w;
        z += sf.f1;
    }
    rc_free(sf);
}
return z;
```

inserted by the compiler!

reference counting

```
struct foo {
    int x1;
    int x2;
};

void f(struct foo *sf);

int x = 1;
int z;
{
    struct foo *sf = rc_alloc(sizeof(struct foo));
    f(sf);
    {
        int w = 1;
        sf.f2 = w;
        z += sf.f1;
    }
    rc_free(sf);
}
return z;
```

reference counting

```
struct foo {
    int x1;
    int x2;
};

void f(struct foo *sf);

int x = 1;
int z;
{
    struct foo *sf = rc_alloc(sizeof(struct foo));
    f(sf);
    {
        int w = 1;
        sf.f2 = w;
        z += sf.f1;
    }
    rc_free(sf);
}
return z;
```

exactly same principle
as **stack allocation**

reference counting

```
struct foo {  
    int x1;  
    int x2;  
};
```

```
struct foo *f() {  
    return rc_alloc(sizeof(struct foo));  
}
```

```
struct foo *sf = f();  
int retval = sf->x1;  
  
return retval;
```

reference counting

```
struct foo {  
    int x1;  
    int x2;  
};
```

```
struct foo *f() {  
    return rc_alloc(sizeof(struct foo));  
}
```

uh-oh...
deallocate?

```
struct foo *sf = f();  
int retval = sf->x1;  
  
return retval;
```

reference counting

```
struct foo {  
    int x1;  
    int x2;  
};
```

```
struct foo *f() {  
    return rc_retain(rc_alloc(sizeof(struct foo)));  
}
```

```
struct foo *sf = f();  
int retval = sf->x1;  
rc_release(sf);  
return retval;
```

reference counting

```
struct foo {  
    int x1;  
    int x2;  
};
```

retain: increment refcount

```
struct foo *f() {  
    return rc_retain(rc_alloc(sizeof(struct foo)));  
}
```

```
struct foo *sf = f();  
int retval = sf->x1;  
rc_release(sf);  
return retval;
```

release: decrement refcount
if zero, free

reference counting

```
struct foo {  
    int x1;  
    int x2;  
};
```

```
struct foo *f() {      rc: 0  
    return rc_retain(rc_alloc(sizeof(struct foo)));  
}                    rc: 1
```

```
    rc: 1  
struct foo *sf = f();  
int retval = sf->x1;  
rc_release(sf); rc: 0    freed!  
return retval;
```

reference counting

```
struct foo {  
    int x1;  
    int x2;  
};
```

```
struct foo *f() {  
    return rc_retain(rc_alloc(sizeof(struct foo)));  
}
```

```
struct foo *sf = f();  
int retval = sf->x1;  
rc_release(sf);  
return retval;
```

in practice: **compiler**
generates retain/release

reference cycles

```
struct foo {
    struct foo *f1;
};

void chain(struct foo *sf1, struct foo *sf2) {
    sf1->f1 = rc_retain(sf2);
    sf2->f1 = rc_retain(sf1);
}

void f() {
    struct foo *sf1 = rc_retain(rc_alloc(sizeof(struct foo)));
    struct foo *sf2 = rc_retain(rc_alloc(sizeof(struct foo)));
    chain(sf1, sf2);
    rc_release(sf1);
    rc_release(sf2);
}

f();
```

reference cycles

```
struct foo {  
    struct foo *f1;  
};
```

```
void chain(struct foo *sf1, struct foo *sf2) {  
    sf1->f1 = rc_retain(sf2);  
    sf2->f1 = rc_retain(sf1);  
}
```

```
void f() {  
    struct foo *sf1 = rc_retain(rc_alloc(sizeof(struct foo)));    rc: 1  
    struct foo *sf2 = rc_retain(rc_alloc(sizeof(struct foo)));    rc: 1  
    chain(sf1, sf2);  
    rc_release(sf1);  
    rc_release(sf2);  
}
```

```
f();
```

reference cycles

```
struct foo {
    struct foo *f1;
};

void chain(struct foo *sf1, struct foo *sf2) {
    sf1->f1 = rc_retain(sf2);    rc: 2
    sf2->f1 = rc_retain(sf1);    rc: 2
}

void f() {
    struct foo *sf1 = rc_retain(rc_alloc(sizeof(struct foo)));    rc: 1
    struct foo *sf2 = rc_retain(rc_alloc(sizeof(struct foo)));    rc: 1
    chain(sf1, sf2);
    rc_release(sf1);
    rc_release(sf2);
}

f();
```

reference cycles

```
struct foo {
    struct foo *f1;
};

void chain(struct foo *sf1, struct foo *sf2) {
    sf1->f1 = rc_retain(sf2);    rc: 2
    sf2->f1 = rc_retain(sf1);    rc: 2
}

void f() {
    struct foo *sf1 = rc_retain(rc_alloc(sizeof(struct foo)));    rc: 1
    struct foo *sf2 = rc_retain(rc_alloc(sizeof(struct foo)));    rc: 1
    chain(sf1, sf2);
    rc_release(sf1);        rc: 1
    rc_release(sf2);        rc: 1
}

f();
```

leaked!

reference cycles

```
struct foo {  
    struct foo *f1;  
};
```

```
void chain(struct foo *sf1, struct foo *sf2) {  
    sf1->f1 = rc_retain(sf2);  
    sf2->f1 = rc_retain(sf1);  
}
```

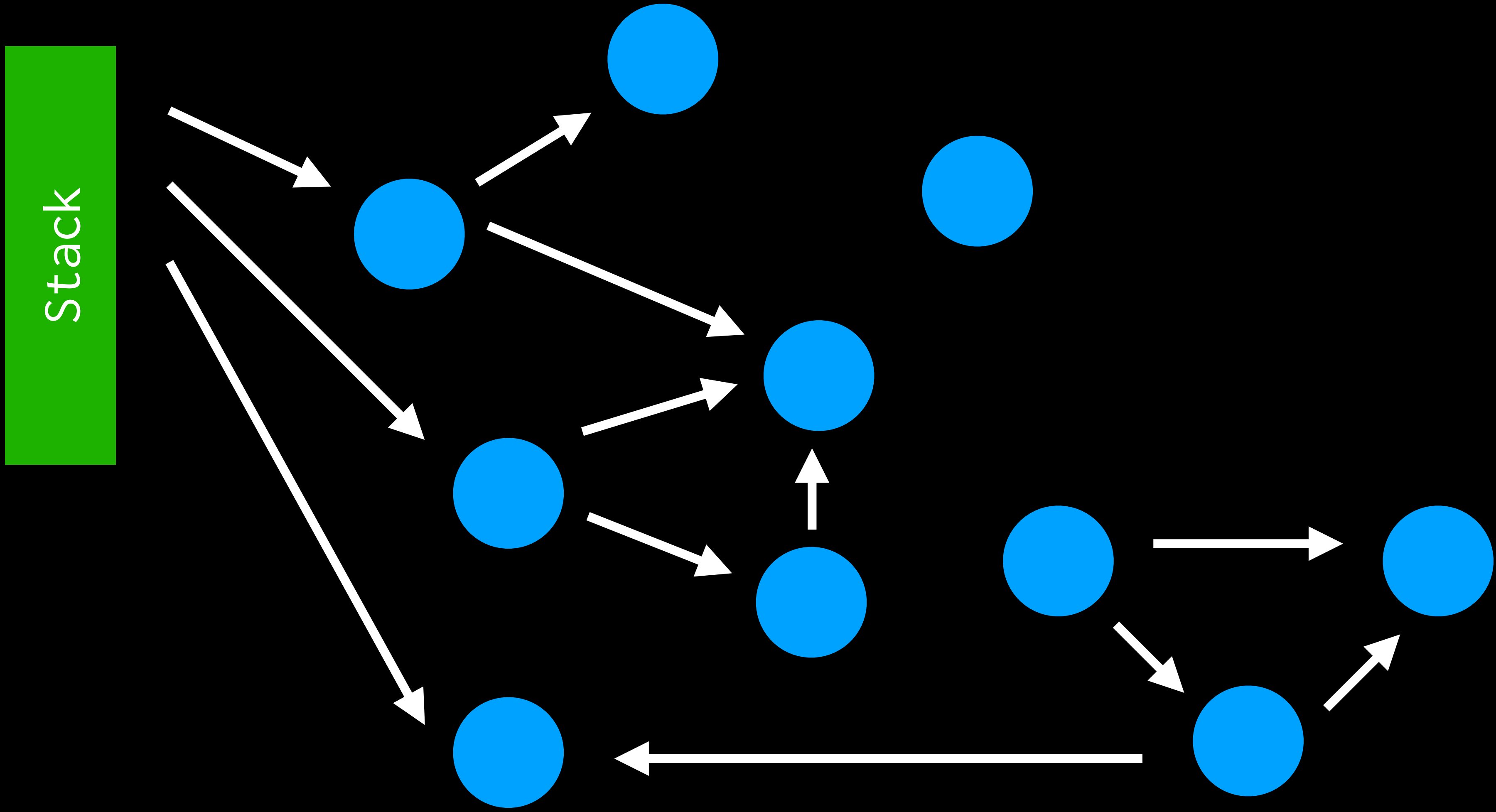
```
void f() {  
    struct foo *sf1 = rc_retain(rc_alloc(sizeof(struct foo)));  
    struct foo *sf2 = rc_retain(rc_alloc(sizeof(struct foo)));  
    chain(sf1, sf2);  
    rc_release(sf1);  
    rc_release(sf2);  
}
```

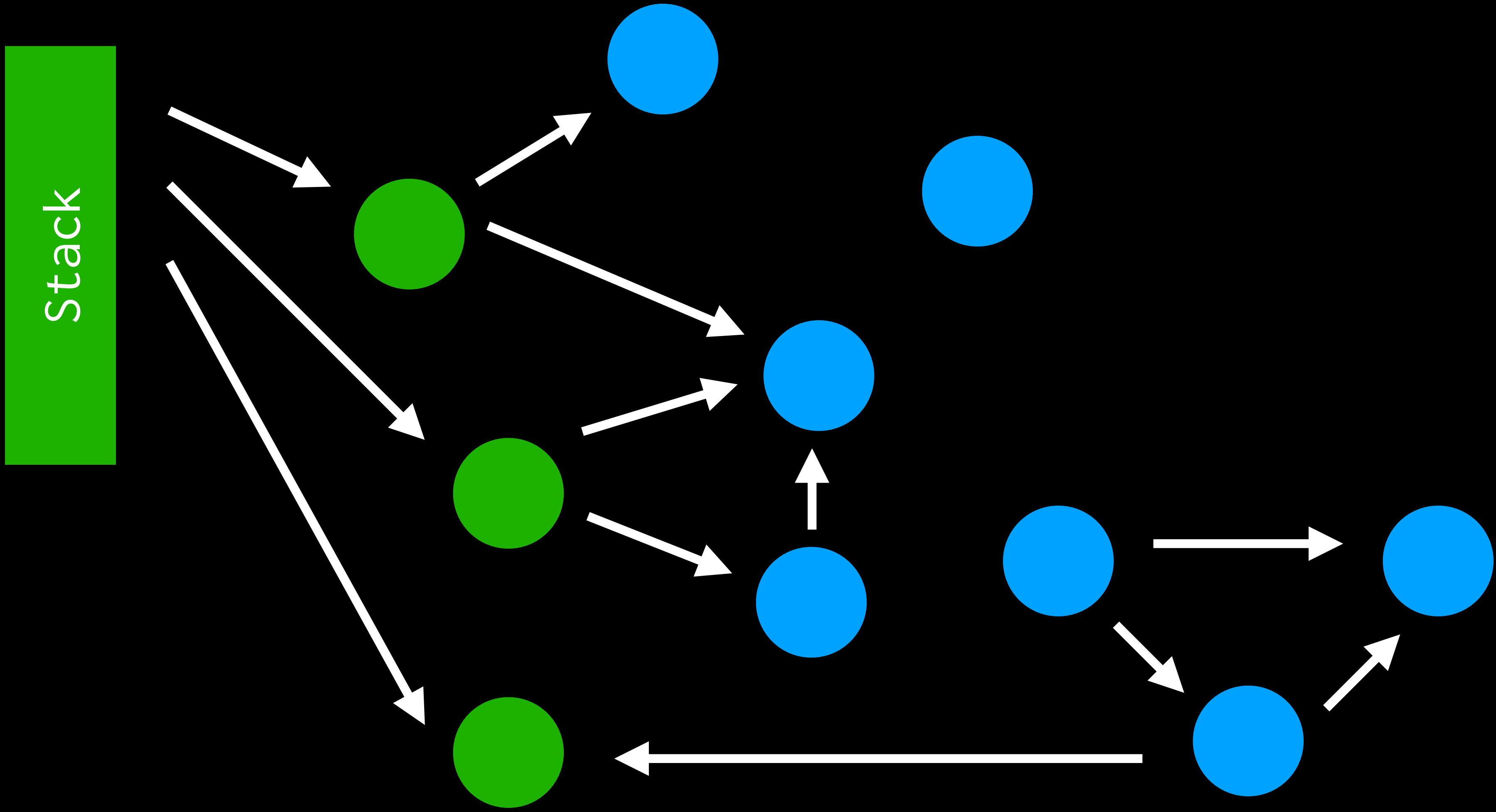
in practice: weak pointers

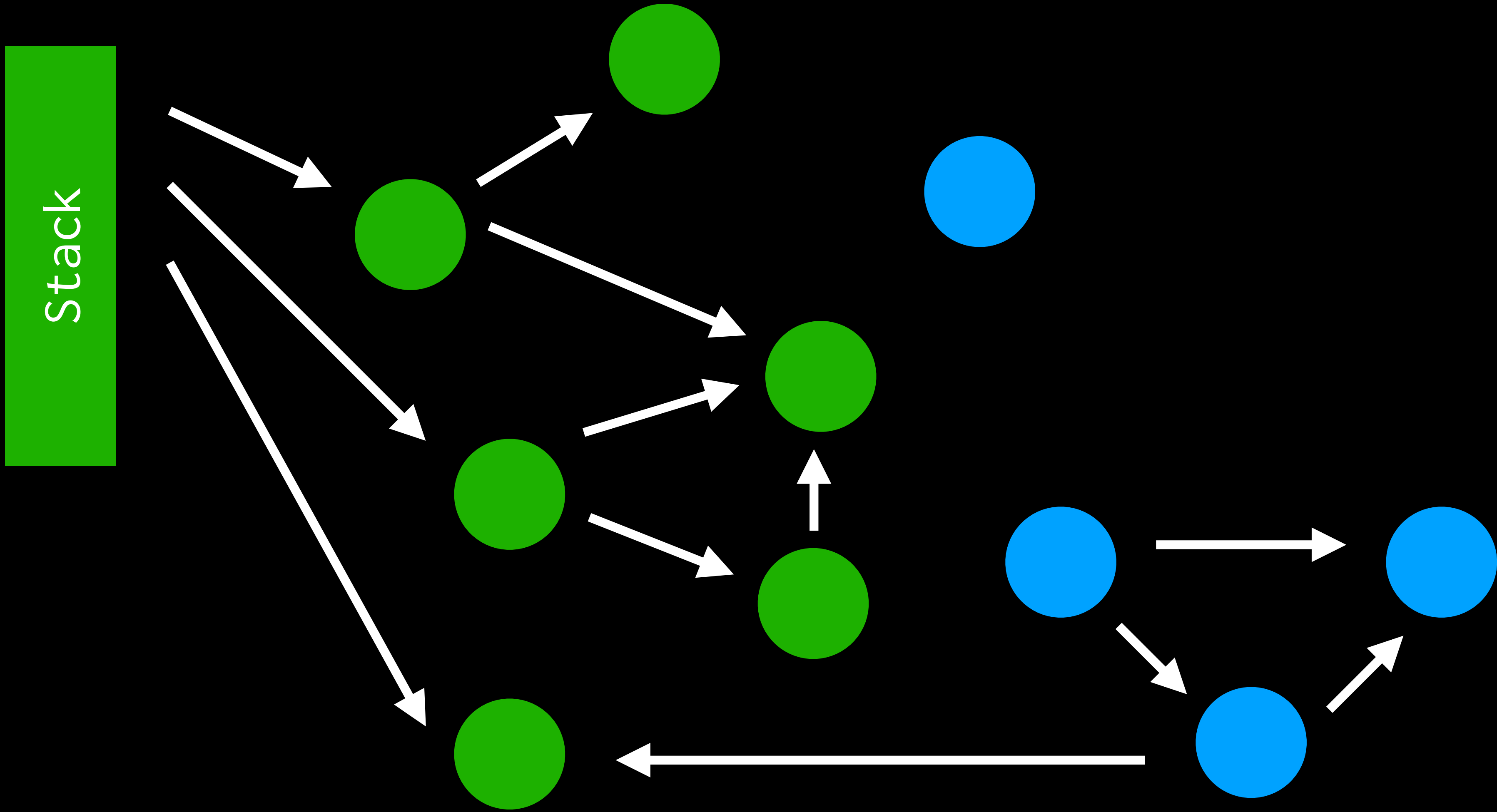
```
f();
```

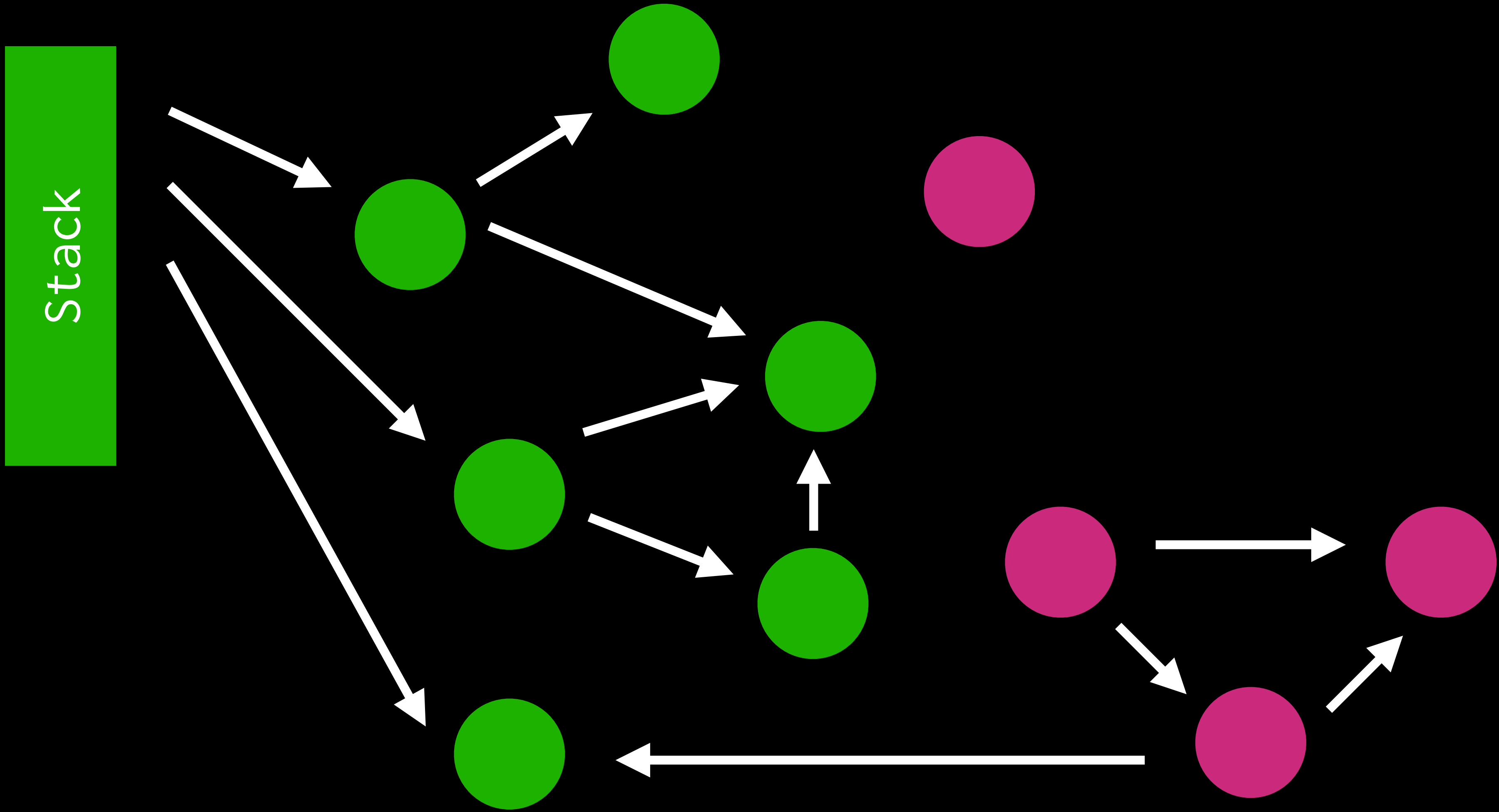
garbage collection

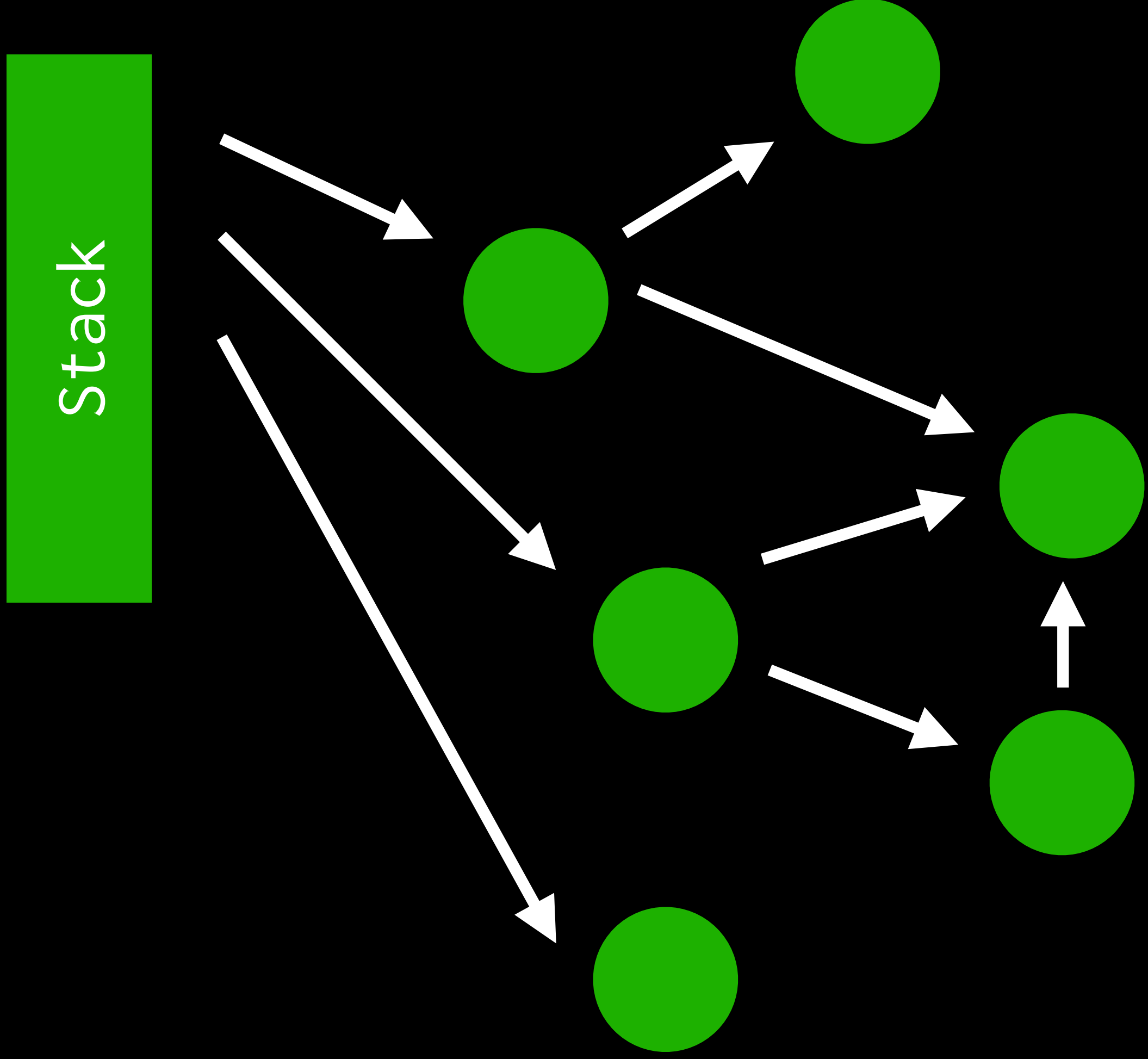
- `traverse heap`, looking for accessible cells
- start at `roots` accessible via stack
- `mark and sweep`: eliminate unaccessible cells
- alternatively, `copy reachable heap`, discard old
- not (directly) based on static analysis













which is better?

garbage collection

- ML, Java, JavaScript, etc.
- free from the **reference cycle** problem
- potentially **compacts** the heap
- other advantages/problems?
- in practice, **generations** and **hybrid schemes**

despite its flaws, the world is GC.

...*but can we do better?*

Q: what made ref counting nontrivial?

A: sharing and cycles.

Q: why is sharing hard?

A: don't **statically** know when to free.

ownership

key idea: *statically* knowing when to free enables
easy, deterministic memory management

ownership

key idea: *statically* knowing when to free enables *easy, deterministic* memory management

GC and RC: *conservative underestimate* of data *ownership* at runtime

ownership

key idea: *statically* knowing when to free enables *easy, deterministic* memory management

GC and RC: *conservative underestimate* of data *ownership* at runtime

delayed free is practically *never!*

ownership

key idea: *statically* knowing when to free enables *easy, deterministic* memory management

GC and RC: *conservative underestimate* of data *ownership* at runtime

delayed free is practically *never!*

stack allocation isn't general...so *generalize* it!

move semantics

each object has a *fixed owner*

sharing of a *unique type* is *statically disallowed*

values can be constructed and destructed, but not *implicitly copied* or *structurally shared*

see: `std::unique_ptr<T>` in C++, `Box<T>` in Rust

can be modeled formally in the *λ -calculus*

weakening

$$\frac{\Gamma_1, \Gamma_2 \vdash e : \tau}{\Gamma_1, x_1 : \tau_1, \Gamma_2 \vdash e : \tau}$$

“useless premises are okay”

contraction

$$\frac{\Gamma_1, x_2 : \tau_1, x_3 : \tau_1, \Gamma_2 \vdash e : \tau_2}{\Gamma_1, x_1 : \tau_1, \Gamma_2 \vdash [x_2/x_1][x_3/x_1]e : \tau_2}$$

“what can be done with many, can be done with one”

linear types

substructural logic lacking *weakening* and *contraction*

like SML with unused variable checking, no sharing

in practice: *affine types* allow *weakening*, not *contraction*


```
val x = []  
val y = 1::x  
val z = x      (* this isn't allowed! *)
```

```
fun clone (xs : 'a list) : ('a list * 'a list) =  
  (xs, xs)
```

```
fun clone (xs : 'a list) : ('a list * 'a list) =  
(xs, xs)
```

sharing isn't allowed

```
fun clone (xs : 'a list) (f : 'a → 'a * 'a) =  
  case xs of  
    [] ⇒ ([], [])  
  | x::xs ⇒  
    let  
      val (x1, x2) = f x  
      val (xs1, xs2) = clone xs f  
    in  
      (x1::xs1, x2::xs2)  
    end
```

why should cloning be **explicit**?

why should cloning be **explicit**?

accurate resource usage

why should cloning be **explicit**?

accurate resource usage

what does cloning a *file* do?

or a *network socket*?

zero allocation

```
fun map (f : 'a → 'b) (xs : 'a list) : 'b list =  
  case xs of  
    [] ⇒ []  
  | x::xs ⇒ (f x)::(map f xs)
```

how many allocations?

zero allocation

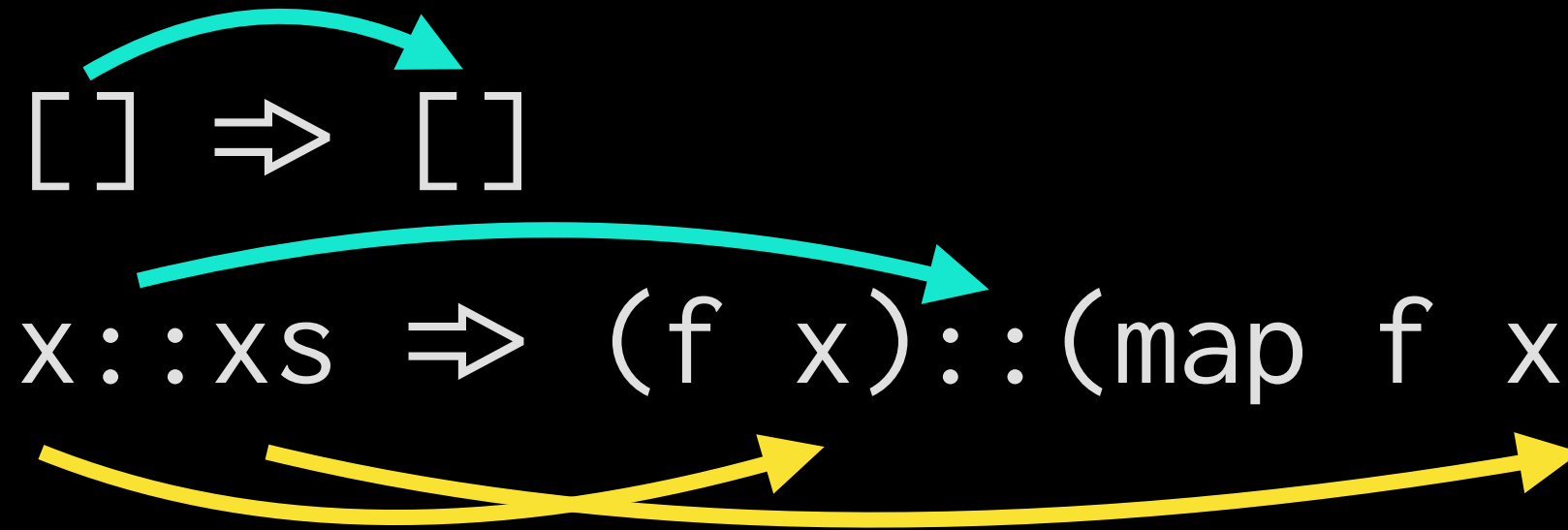
linear type!

```
fun map (f : 'a → 'b) (xs : 'a list) : 'b list =
```

```
  case xs of
```

```
    [] ⇒ []
```

```
  | x :: xs ⇒ (f x) :: (map f xs)
```



zero allocation

```
fun map (f : 'a → 'b) (xs : 'a list) : 'b list =
```

```
  case xs of
```

```
    [] ⇒ []
```

```
  | x::xs ⇒ (f x)::(map f xs)
```

no allocations!

optimize to an
in-place for-loop!

```
fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val table = List.fold (fn ((i, x), acc) =>
      let
        val existing = case Table.get acc i of
          NONE => []
          | SOME l => l
        val new = x::existing
      in
        Table.set acc i new
      end
    ) Table.empty xs
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end
```

```

fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val table = List.fold (fn ((i, x), acc) =>
      let
        val existing = case Table.get acc i of
          NONE => []
          | SOME l => l
        val new = x::existing
      in
        Table.set acc i new
      end
    ) Table.empty xs
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end

```

no sharing!

```
fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val table = List.fold (fn ((i, x), acc) =>
      let
        val (existing, acc') = case Table.remove acc i of
          (NONE, acc') => ([], acc')
          | (SOME l, acc') => (l, acc')
        val new = x::existing
      in
        Table.add acc' i new
      end
    ) Table.empty xs
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end
```

```
fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val table = List.fold (fn ((i, x), acc) =>
      let
        val (existing, acc') = case Table.remove acc i of
          (NONE, acc') => ([], acc')
          | (SOME l, acc') => (l, acc')
        val new = x::existing
      in
        Table.add acc' i new
      end
    ) Table.empty xs
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end
```

wow, that's annoying

```
fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val table = Table.empty ()
    val _ = List.fold (fn ((i, x), _) =>
      let
        val existing = case Table.delete table i of
          NONE => []
          | SOME l => l
        val new = x::existing
      in
        Table.insert table i new
      end
    ) xs ()
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end
```

Q: isn't imperative programming bad?

A: well, observable side effects, maybe.


```
val x = ref 1
val y = x
val z = y := 2
val 1 = !x      (* exception Bind *)
```

ML world

```
val x = ref 1
```

```
val y = x
```

```
val z = y := 2
```

```
val 1 = !x
```

Error: 'x' used after move

linear world

```
let l = [1, 2, 3];    let l = [1, 2, 3];  
l.push(4);           let l = l @ [4];
```

imperative = functional?

“Linear types can change the world!”

Phil Wadler, 1990

<http://www.cs.ioc.ee/ewscs/2010/mycroft/linear-2up.pdf>

```
fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val table = Table.empty ()
    val _ = List.fold (fn ((i, x), _) =>
      let
        val existing = case Table.remove table i of
          NONE => []
          | SOME l => l
        val new = x::existing
      in
        Table.insert table i new
      end
    ) xs ()
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end
```

```

fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val table = Table.empty ()
    val _ = List.fold (fn ((i, x), _) =>
      let
        val existing = case Table.remove table i of
          NONE => []
          | SOME l => l
        val new = x::existing
      in
        Table.insert table i new
      end
    ) xs ()
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end

```

```

fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val table = Table.empty ()
    val _ = List.fold (fn ((i, x), _) =>
      let
        val existing = case Table.remove table i of
          NONE => []
          | SOME l => l
        val new = x::existing
      in
        Table.insert table i new
      end
    ) xs ()
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end

```

really just state monad?

```
fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val table = Table.empty ()
    val _ = List.fold (fn ((i, x), _) =>
      let
        val existing = case Table.remove table i of
          NONE => []
          | SOME l => l
        val new = x::existing
      in
        Table.insert table i new
      end
    ) xs ()
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end
```

really just a *binding*

references

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash \&e : \&\tau}$$

references are **exempt** from linearity

construction: tied to owned object

destruction: freely before owned object

```
fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val table = Table.empty ()
    val _ = List.fold (fn ((i, x), _) =>
      let
        val existing = case Table.remove (&table) i of
          NONE => []
          | SOME l => l
        val new = x::existing
      in
        Table.insert (&table) i new
      end
    ) xs ()
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end
```

```

fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val table = Table.empty ()
    val _ = List.fold (fn ((i, x), _) =>
      let
        val existing = case Table.remove (&table) i of
          NONE => []
          | SOME l => l
        val new = x::existing
      in
        Table.insert (&table) i new
      end
    ) xs ()
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end

```

predictable resource usage
+ imperative efficiency
+ almost no observable side effects

```
fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val table = Table.empty ()
    val _ = List.fold (fn ((i, x), _) =>
      let
        val existing = case Table.remove (&table) i of
          NONE => []
          | SOME l => l
        val new = x::existing
      in
        Table.insert (&table) i new
      end
    ) xs ()
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end
```

what if someone held (&table) here?



```
fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val table = Table.empty ()
    val _ = List.fold (fn ((i, x), _) =>
      let
        val existing = case Table.remove (&table) i of
          NONE => []
          | SOME l => l
        val new = x::existing
      in
        Table.insert (&table) i new
      end
    ) xs ()
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end
```

what if someone held (&table) here?
possible race condition!

data race freedom

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash \&\text{mut } e : \&\text{mut } \tau}$$

mutable references are **not** (fully) **exempt** from linearity

construction: *only one* per object,
and not when any *immutable* refs exist!

```
fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val mut table = Table.empty ()
    val _ = List.fold (fn ((i, x), _) =>
      let
        val existing = case Table.remove (&mut table) i of
          NONE => []
          | SOME l => l
        val new = x::existing
      in
        Table.insert (&mut table) i new
      end
    ) xs ()
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end
```

+ no observable side effects

```
fun aggregate (xs : (int * 'a) list) : (int * 'a list) list =
  let
    val mut table = Table.empty ()
    val _ = List.fold (fn ((i, x), _) =>
      let
        val existing = case Table.remove (&mut table) i of
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        val new = x::existing
      in
        Table.insert (&mut table) i new
      end
    ) xs ()
  in
    Table.fold (fn ((i, xs), acc) => (i, xs)::acc) [] table
  end
```

Rust, in a nutshell!

regions

- how long can references last?

regions

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regions

- how long can references last?
- **Swift**: “as long as the ref count is above zero”
- **ML**: “as long as the object is reachable”
- **C**: “who knows?”
- **Rust**:
a reference `&'a T` lasts as long as the *lifetime* `'a`

regions

- Ownership requires explicit allocation
- References must not outlast values
- Can ownership be inferred?
- Generalizing stack allocation fully: region types

regions

- Annotate values with region variables
- Lambdas form region closures
- Evaluation destroys regions
- Type safety: no dangling pointer is dereferenced
- Region annotation: meaning-preserving refinement
- At runtime: stack allocation, bump allocators, deterministic whole region deallocation
- Effect types

```
{ region <'r1> r1;
  list_t<int,'r1> x = NULL;
  { region <'r2> r2;
    list_t<int,'r2> y = rcopy(r2, x);
    x = rnew(r1) List(4,x);
    y = rnew(r2) List(5,y);
    // x = rnew(r2) List(4,x); // Error : incompatible regions
    // x = rnew(r1) List(4,y); // Error : List in unique region
  }
}
```

Cyclone


```
list_t<int, 'r> fib_rec(region_t<'r> r, int a, int b, int n)
{
    if (n == 0)
        return rnew(r) List(b, NULL);
    else {
        region <'s> s;
        list_t<int, 's> z = fib_rec(s, b, a+b, n-1);
        z = rnew(s) List(b, z);
        return rcopy(r, z);
    }
}

list_t<int, 'r> fib(region_t<'r> r, int n) {
    return fib_rec(r, 1, 1, n-1);
}
```

Cyclone

in practice

- **C++**: uniqueness and reference counting
- **Rust**: affine types, unique by default, lifetimes
- **Cyclone**: LIFO regions, heap and stack regions
- **ML Kit**: viable alternative to universal GC for SML
- **Clean**: uniqueness types
- **Idris**: uniqueness and dependent types

further reading

ch. 1 (linear types),
ch. 3 (effect and region types) of
Advanced Topics in Types and Programming Languages (by Pierce)

“Linear types can change the world!”, Wadler, 1990.

“Typed memory management in a calculus of capabilities”,
Crary et. al., POPL '99.

[http://www.labri.fr/perso/renault/working/research/files/
rust2016.2.pdf](http://www.labri.fr/perso/renault/working/research/files/rust2016.2.pdf)