Introduction

Welcome to Hype for Types!

- Instructors:
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  - Isabel Gan (igan)
- Attendance
  - In general, you have to come to lecture to pass
  - Let us know if you need to miss a week
- Homework
  - Every lecture will have an associated homework
  - Graded on effort (*not* correctness)
  - If you spend more than an hour, please stop[1]

[1] Unless you're having fun!

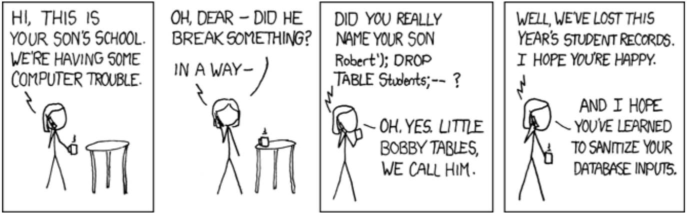
Other Stuff

- Please join the Discord and Gradescope if you haven't
- We assume everyone has 150 level knowledge of functional programming and type systems
  - If you don't have this and feel really lost, talk to us after class

Motivation

Programming is Hard

There are many common classes of mistakes/bugs/errors in code:



`https://xkcd.com/327/`

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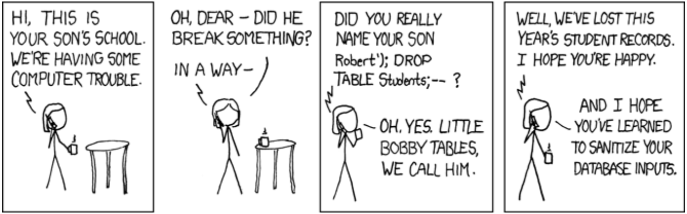

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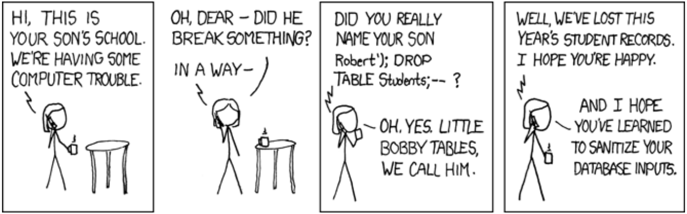

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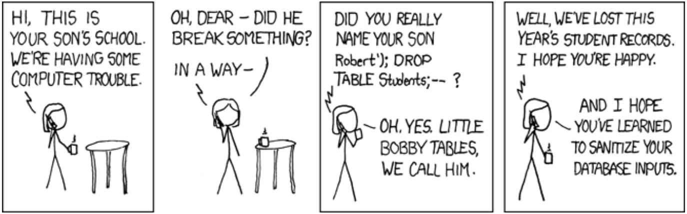

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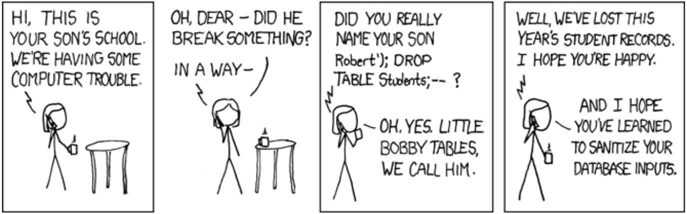

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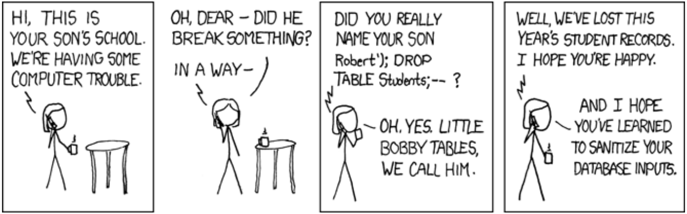

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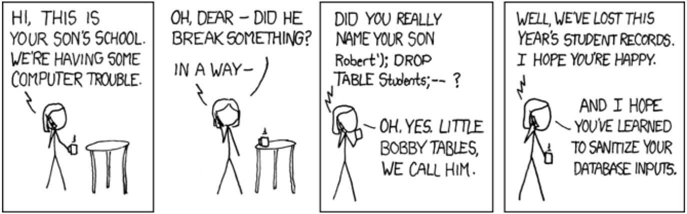
- `A[len(A)]`
- `@requires is_sorted(A)`



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Types are... hype!

Types are *descriptions* of how some piece of data can be used.

[2] Foreshadowing: "a literary device in which a writer gives an advance hint of what is to come later in the story." *Wikipedia, "Foreshadowing," retrieved 30 Aug 2022*

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Guiding Question

How can we use types to catch errors at compile-time?

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Can we use types for more than just bug-catching?[2]

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Lambda Calculus

Building a tiny language

The *simply-typed lambda calculus* is simple. It only has four features:

- Unit ("empty tuples")
- Booleans
- Tuples
- Functions

Expressions

We represent our expressions using a *grammar*:

e	$::=$	x	variable
	$\mid$	$\langle\rangle$	unit
	$\mid$	$\textbf{false}$	false boolean
	$\mid$	$\textbf{true}$	true boolean
	$\mid$	$\textbf{if}\ e_1\ \textbf{then}\ e_2\ \textbf{else}\ e_3$	boolean case analysis
	$\mid$	$\langle e_1, e_2\rangle$	tuple
	$\mid$	$\textbf{fst}(e)$	first tuple element
	$\mid$	$\textbf{snd}(e)$	second tuple element
	$\mid$	$\lambda x : \tau.\ e$	function abstraction (lambda)
	$\mid$	$e_1\ e_2$	function application

Types

Similarly, we define our types as follows:

$$\begin{array}{rcl} \tau & ::= & \textbf{unit} \\ & | & \textbf{bool} \\ & | & \tau_1 \times \tau_2 \\ & | & \tau_1 \rightarrow \tau_2 \end{array}$$

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Question

How do we check if $e : \tau$?

Inference Rules

In logic, we use *inference rules* to state how facts follow from other facts.

$$\frac{\text{premise}_1 \quad \text{premise}_2 \quad \dots}{\text{conclusion}}$$

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For example:

$$\frac{\text{you are here} \quad \text{you are hyped}}{\text{you are hyped for types}}$$

$$\frac{}{\text{functions are values}}$$

$$\frac{\text{it's raining} \quad x \text{ is outside}}{x \text{ is getting wet}}$$

$$\frac{A \text{ ancestor } B \quad B \text{ mother } C}{A \text{ ancestor } C}$$

$$\frac{n \text{ is a number}}{n+1 \text{ is a number}}$$

$$\frac{f \text{ total} \quad x \text{ valuable}}{f\ x \text{ valuable}}$$

Typing Rules: First Attempt

Consider the judgement $e : \tau$ ("e has type τ"). Let's try to express some simple typing rules.

$$\frac{}{\langle\rangle : \textbf{unit}} \qquad \frac{}{\textbf{false} : \textbf{bool}} \qquad \frac{}{\textbf{true} : \textbf{bool}} \qquad \frac{e_1 : \textbf{bool} \quad e_2 : \tau \quad e_3 : \tau}{\textbf{if } e_1 \textbf{ then } e_2 \textbf{ else } e_3 : \tau}$$

$$\frac{e_1 : \tau_1 \quad e_2 : \tau_2}{\langle e_1, e_2 \rangle : \tau_1 \times \tau_2} \qquad \frac{e : \tau_1 \times \tau_2}{\textbf{fst}(e) : \tau_1} \qquad \frac{e : \tau_1 \times \tau_2}{\textbf{snd}(e) : \tau_2}$$

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Question

How do we write rules for functions?

Typing Rules: Functions

Let's give it a shot.

$$\frac{e_1 : \tau_1 \to \tau_2 \quad e_2 : \tau_1}{e_1\ e_2 : \tau_2}$$

Looks good so far...

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$$\frac{e : \tau_2\ \textcolor{red}{(?)}}{\lambda x : \tau_1.\ e : \tau_1 \to \tau_2}$$

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Key Idea

Expressions only have types *given a context*!

Contexts

Intuition

If, given $x : \tau_1$, we know $e : \tau_2$, then $\lambda x : \tau_1 .\ e : \tau_1 \to \tau_2$.

Therefore, we need a context (denoted Γ) which associates types with variables.

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1 .\ e : \tau_1 \to \tau_2}$$

What types does some variable x have? It depends on the previous code!

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau}$$

All the rules!

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \text{ (VAR)} \qquad \frac{}{\Gamma \vdash \langle\rangle : \textbf{unit}} \text{ (UNIT)} \qquad \frac{}{\Gamma \vdash \textbf{false} : \textbf{bool}} \text{ (FALSE)}$$

$$\frac{}{\Gamma \vdash \textbf{true} : \textbf{bool}} \text{ (TRUE)} \qquad \frac{\Gamma \vdash e_1 : \textbf{bool} \quad \Gamma \vdash e_2 : \tau \quad \Gamma \vdash e_3 : \tau}{\Gamma \vdash \textbf{if } e_1 \textbf{ then } e_2 \textbf{ else } e_3 : \tau} \text{ (IF)}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash \langle e_1, e_2 \rangle : \tau_1 \times \tau_2} \text{ (TUP)} \qquad \frac{\Gamma \vdash e : \tau_1 \times \tau_2}{\Gamma \vdash \textbf{fst}(e) : \tau_1} \text{ (FST)}$$

$$\frac{\Gamma \vdash e : \tau_1 \times \tau_2}{\Gamma \vdash \textbf{snd}(e) : \tau_2} \text{ (SND)} \qquad \frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1 .\ e : \tau_1 \to \tau_2} \text{ (ABS)}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \to \tau_2 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1\ e_2 : \tau_2} \text{ (APP)}$$

Example: what's the type?

Let's derive that

$$\cdot \vdash (\lambda x : \textbf{unit}.\ \langle x, \textbf{true} \rangle)\ \langle\rangle : \textbf{unit} \times \textbf{bool}$$

by using the rules.

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Homework Foreshadowing

That looks like a trace of a typechecking algorithm!

Get Hype.

The Future is Bright

- How can you use basic algebra to manipulate types?
- How do types and programs relate to logical proofs?
- How can we automatically fold (and unfold) any recursive type?
- How can types allow us to do safe imperative programming?
- Can we make it so that programs that typecheck iff they're correct?