

Please choose (at least) one of the following problems to submit.

1. Define `isSorted` by

```
isSorted [] = 1
isSorted [x] = 1
isSorted x :: y :: xs = if x ≤ y andalso isSorted y :: xs then 1 else 0
```

Write dependently typed pseudocode (anything is OK as long as it's reasonable) with type

$$[\mathbb{N}] \rightarrow \Sigma_{xs: [\mathbb{N}]} \text{isSorted}(xs)$$

2. Two types  $A$  and  $B$  are said to be **isomorphic** if there exist functions  $f : A \rightarrow B$  and  $g : B \rightarrow A$  such that  $f \circ g = id$  and  $g \circ f = id$ .

Show that

$$\Sigma_{x:A} \Sigma_{y:B(x)} C((x, y)) \cong \Sigma_{z:\Sigma_{x:A} B(x)} C(z)$$

3. Using the definition of  $>$  from the notes, prove

$$\prod_{n:\mathbb{N}} ((n = 0) + (n > 0))$$

by finding a value of the correct type.