Functional Programming & Constructive Logic

Hype for Types, Lecture 3

Password: constructive

Logic

^	F	T
F	F	F
T	F	T

A => B == -A v B

-(A v B) == (-A ^ -B)

…

=>	F	T
F	T	T
T	F	T

v	F	T
F	F	T
T	T	T

$\oplus$	F	T
F	F	T
T	T	F

- Nothing new
- 151/127/128 flashbacks

What if we made logic

better

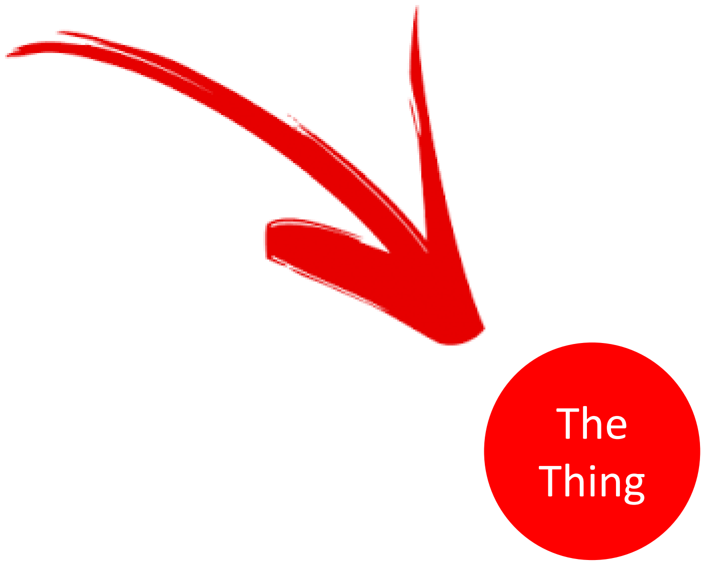
Take logic, and make one

sensible

addition

A sensible statement

"If you wish to prove to me that something exists, you must show me the thing."



Examples

- Do unicorns exist?
  - Proof 1: Here is a unicorn:

  - Proof 2: Assume unicorns did not exist. Then, no magical animal would ride on rainbows. But we know there is a magical animal that rides on rainbows. Contradiction! Therefore, unicorns exist.

Examples

- Did you do your homework?
  - Proof 1: Here is my homework submission:

    1 - jluningp@andrew.cmu.edu_1_handin.sml

  - Proof 2: Assume you did not do your homework. Then we would be sad. But we are not sad. Contradiction! Therefore, you did your homework.

Examples

- Can you write code that parses lambda calculus?
  - Proof 1: Here is a parser

```
sml
name LambdaCalculusParseFun

terminal IDENT of string

terminal LPAREN
terminal RPAREN

terminal LAMBDA
terminal DARROW

nonterminal Exp : exp =
  1:ExpOne => id_exp
  1:Exp 2:ExpOne => Apply

nonterminal ExpOne : exp =
  1:IDENT => Variable
  LPAREN 1:Exp RPAREN => id_exp
  LAMBDA 1:IDENT DARROW 2:Exp => Lambda

start Exp
```

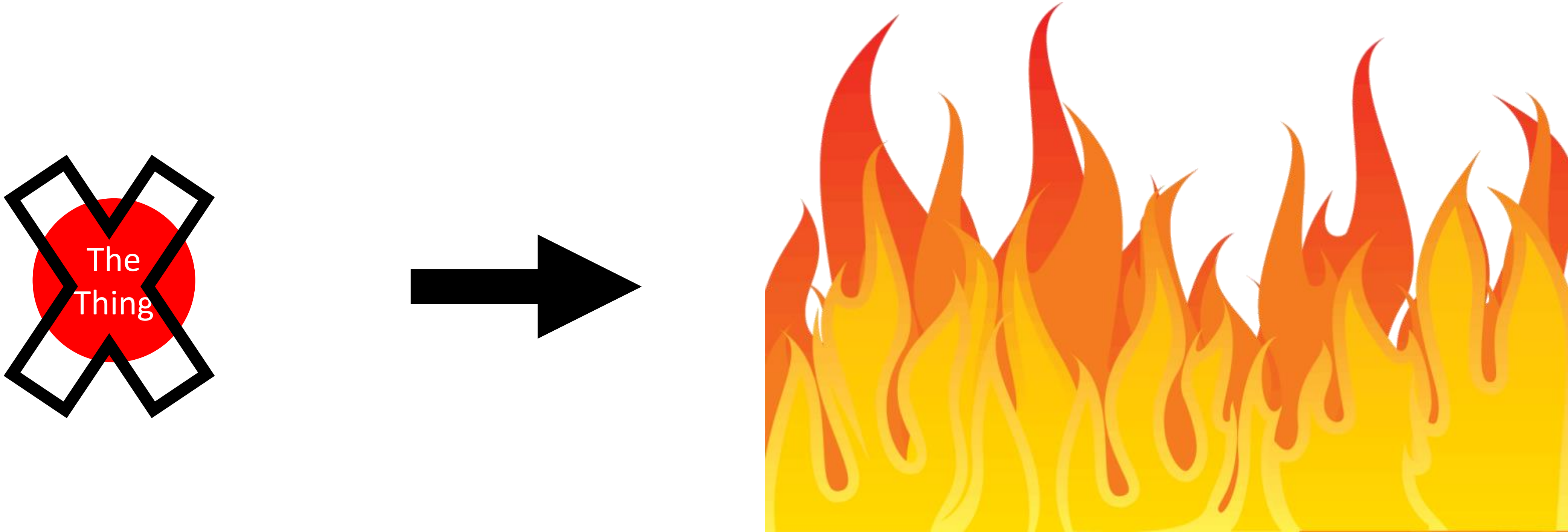
  - Proof 2: Assume you can’t. [A couple properties of lambda calculus later]. Contradiction! Therefore, you can parse lambda calculus.

The difference between Proof 1 and Proof 2 is information content.

- Do unicorns exist?
  - Proof 1: I have a *unicorn* now.
  - Proof 2: uh…
- Did you do your homework?
  - Proof 1: I can now find and grade your homework.
  - Proof 2: uh…
- Can you write code that parses lambda calculus?
  - Proof 1: If I install cmyacc, I can parse lambda calculus now.
  - Proof 2: uh…

Another statement

"It is *not* enough to prove that *bad things* will happen if the thing did not exist."



Constructive Logic

- The logic of “If you wish to prove that something exists, you must produce the thing.”
- The logic of type theory, programming language theory, and computer science in general.
- Our namesake: 15-317

Now for some inference rules

Typechecking rules from last week

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau} \qquad \frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \texttt{fn}\ (x : \tau_1) \Rightarrow e : \tau_1 \to \tau_2} \qquad \frac{\Gamma \vdash e' : \tau_1 \quad \Gamma \vdash e : \tau_1 \to \tau_2}{\Gamma \vdash e\ e' : \tau_2}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash (e_1, e_2) : \tau_1 \times \tau_2} \qquad \frac{\Gamma \vdash e : \tau_1 \times \tau_2}{\Gamma \vdash \#1\ e : \tau_1} \qquad \frac{\Gamma \vdash e : \tau_1 \times \tau_2}{\Gamma \vdash \#2\ e : \tau_2}$$

$$\frac{\Gamma \vdash e : \tau_1}{\Gamma \vdash \texttt{INL}\ e\ \texttt{into}\ \tau_1 + \tau_2 : \tau_1 + \tau_2} \qquad \frac{\Gamma \vdash e : \tau_2}{\Gamma \vdash \texttt{INR}\ e\ \texttt{into}\ \tau_1 + \tau_2 : \tau_1 + \tau_2}$$

$$\frac{\Gamma \vdash e : \tau_1 + \tau_2 \quad \Gamma, x_1 : \tau_1 \vdash e_1 : \tau \quad \Gamma, x_2 : \tau_2 \vdash e_2 : \tau}{\Gamma \vdash \texttt{case}\ e\ \texttt{of}\ \texttt{INL}\ x_1 \Rightarrow e_1\ \texttt{|}\ \texttt{INR}\ x_2 \Rightarrow e_2 : \tau}$$

Why constructive?

- In programming, we must actually compute and construct things.

- When I prove something using code, that code actually computes the proof
  - Example:

$$(\text{fn } (x : A) \Rightarrow M) : A \to B$$

  is a function from a proof of A to a proof of B. If I call this function, it actually has to produce the proof of B.