Monads II: Theory & Applications

Hype for Types

March 23, 2026

Review: The Pattern of "And Then"

Recap: The Core Abstraction

The Payoff

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The SML Signature

```
signature MONAD = sig
  type 'a t
  val return : 'a -> 'a t
  val bind   : 'a t * ('a -> 'b t) -> 'b t
end
```

The Laws: Why "Trust" Matters

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The three laws are:

1. **Left Identity**
2. **Right Identity**
3. **Associativity**

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Breaking the Law: The Loggy Return

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Now, `return 5 >>= f` prints a message, but `f 5` does not.

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The Problem: You can't refactor! You can no longer replace a redundant `return` with a direct function call without changing the program's output (the logs).

Law 2: Right Identity

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The Problem: This ruins **Dead Code Elimination**. A compiler should be able to remove `x >>= return` if the result isn't used. If laws are broken, the compiler accidentally changes your program logic.

Law 3: Associativity

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Breaking the Law

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The Problem: This ruins **Composition**. Associativity is what allows us to write flat `let%bind` or `do` blocks without worrying about nesting.

The Theory: Kleisli Arrows

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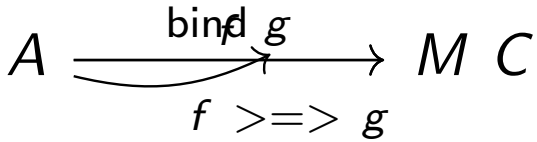

The State Monad

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  let
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Monadic Insight: A stateful computation is just a function: `'s -> ('a * 's)`.

Concurrency vs. Parallelism

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Analogy: The Video Project

- **Exporting Video (CPU-bound)**: Hard math. Your CPU is at 100%. You need **Parallelism** to speed this up.
- **Downloading Video (I/O-bound)**: Your CPU is at 0%. It's sitting idle waiting for the network. You need **Concurrency** to do other work while you wait.

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If 1 CPU instruction (reading a register) took **1 second**:

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The Goal: Instead of the CPU sitting idle for "4 months," we want a way to **pause** a function, save its state, and **resume** it when the mail arrives.

Async/Await — Concurrency as a Monad

Future¡T¿ is a Monad

In Rust, a `Future<T>` is a value that will eventually become a `T`.

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The Monadic Mapping

- `return`: The `async { }` block. It wraps a value in a Future that is "Ready."
- `bind`: The `.await` keyword. It says: "Pause this function. When the Future is ready, take the inner value and continue."

The Type System as a State Machine

How does Rust "remember" where it was when it pauses? The compiler transforms your function into an **enum-based state machine**.

```
// Your Code:
let user = fetch_user(id).await;
let post = fetch_post(user.id).await;

// The Compiler's Transformation (Simplified):
enum MyFunctionState {
    Start { id: u32 },
    WaitingForUser { fut: FetchUserFuture },
    WaitingForPost { user: User, fut: FetchPostFuture },
    Done
}
```

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Payoff

The "paused" state is a **Type**. The compiler calculates exactly how much memory it needs. Because it's a Monad, the type system can statically track exactly which functions can "pause" and which cannot.

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- **Effect Visibility**: You cannot accidentally call a months-long network request inside a high-speed synchronous loop. The types won't match!
- **No Runtime Overhead**: Because the State Machine is just an enum, there is no need for heavy "Green Threads" or hidden stack-copying.

What's Next

Next Week: The SML Bee!

High-speed, typed competition. Review your Bind, your Return, and your pattern matching.

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No lecture. Just code.